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TEKISLIKLAR VA TO‘G‘RI CHIZIQLARNING O‘ZARO JOYLASHUVI

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Annotatsiya

Ushbu maqolada fazoda va tekislikda tekisliklar hamda to‘g‘ri chiziqlarning o‘zaro joylashuvi masalalari o‘rganilgan. Tekisliklar va to‘g‘ri chiziqlar orasidagi burchaklarni aniqlash, ularning parallel, perpendikulyar, kesishuvchi va ayqash holatlari tahlil qilingan. Shuningdek, nuqtadan tekislikka va to‘g‘ri chiziqqacha bo‘lgan masofalarni hisoblash usullari, yo‘naltiruvchi kosinuslar, proyeksiyalar hamda simmetrik nuqtalarni topish formulalari yoritilgan. Nazariy ma’lumotlar amaliy misollar yordamida mustahkamlanib, analitik geometriyaning muhim masalalarini yechish usullari ko‘rsatib berilgan.

Kalit so‘zlar

Tekislik, to‘g‘ri chiziq, normal vektor, yo‘naltiruvchi vektor, burchak, perpendikulyarlik.

Kirish. Tekisliklar $A_1x + B_1y + C_1 = 0$ va $A_2x + B_2y + C_2 = 0$ tenglamalar bilan berilgan bo‘lsin. Ular orasidagi φ burchak quyidagi formula asosida hisoblanadi: $\cos\varphi = \frac{\overrightarrow{n_1}\cdot\overrightarrow{n_2}}{|\overrightarrow{n_1}|\cdot|\overrightarrow{n_2}|} = \frac{A_1A_2+B_1B_2+C_1C_2}{\sqrt{{A_1}^2+{B_1}^2+{C_1}^2}\cdot\sqrt{{A_2}^2+{B_2}^2+{C_2}^2}}$ bunda

$\overrightarrow{n_1} = \{A_1, B_1, C_1\}$ va $\overrightarrow{n_2} = \{A_2, B_2, C_2\}$ berilgan tekisliklarning normal vektorlari.

a) Agar tekisliklar perpendikulyar bo‘lsa, u holda $\overrightarrow{n_1}\cdot\overrightarrow{n_2} = 0\ yoki$ $A_1A_2 + B_1B_2 + C_1C_2$=0.

b) Agar tekisliklar parallel bo‘lsa, u holda $\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2} \neq \frac{D_1}{D_2}$

c) Agar tekisliklar ustma-ust tushsa, u holda $\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2} = \frac{D_1}{D_2}$

d) $M_0(x_0, y_0, z_0)\ nuqtadan\ Ax + By + Cz + D = 0$ tekislikkacha bo‘lgan d masofa: $d = \frac{|Ax_0+By_0+Cz_0+D|}{\sqrt{A^2+B^2+C^2}}\ formula\ bo'yicha\ hisoblanadi.$ To‘g‘ri chiziqlar $\frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{p_1}$ va $\frac{x-x_2}{l_2} = \frac{y-y_2}{m_2} = \frac{z-z_2}{p_2}$ kanonik tenglamalar bilan berilgan bo‘lsin. Bu to‘g‘ri chiziqlar orasidagi φ burchak formuladan topiladi:

$$\cos\varphi = \frac{\overrightarrow{s_1}\cdot\overrightarrow{s_2}}{|\overrightarrow{s_1}|\cdot|\overrightarrow{s_2}|} = \frac{l_1l_2+m_1m_2+p_1p_2}{\sqrt{{l_1}^2+{m_1}^2+{p_1}^2}\cdot\sqrt{{l_2}^2+{m_2}^2+{p_2}^2}}$$

a) Agar to‘g‘ri chiziqlar perpendikulyar bo‘lsa, u holda $\overrightarrow{s_1}\cdot\overrightarrow{s_2}=0$ yoki $l_1l_2+m_1m_2+p_1p_2=0$.

b) Agar to‘g‘ri chiziqlar parallel bo‘lsa, u holda

$$\frac{l_1}{l_2}=\frac{m_1}{m_2}\neq\frac{p_1}{p_2}$$

c)Agar to‘g‘ri chiziqlar ustma-ust tushsa, u holda

$$\frac{l_1}{l_2}=\frac{m_1}{m_2}=\frac{p_1}{p_2}$$

shu bilan birga $\frac{x_2-x_1}{l_1}=\frac{y_2-y_1}{m_1}=\frac{z_2-z_1}{p_1}$

d)Agar to‘g‘ri chiziqlar kesishsa, u holda $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & p_1 \\ l_2 & m_2 & p_2 \end{vmatrix}=0$

e)Agar to‘g‘ri chiziqlar ayqash bo‘sa, u holda $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & p_1 \\ l_2 & m_2 & p_2 \end{vmatrix}\neq 0$

$M_1(x_1,y_1,z_1)\ nuqtadan\ \ \frac{x-x_0}{l}=\frac{y-y_0}{m}=\frac{z-z_0}{p}$ to‘g‘ri chiziqqacha bo‘lgan masofa quyidagi formula bo‘yicha hisoblanadi: $d=\frac{|\vec{s}\times\overrightarrow{M_1M_0}|}{|\vec{s}|}$ bunda $M_0(x_0,y_0,z_0)$ nuqta shu to‘g‘ri chiziqqa tegishli va $\vec{s}=\{l,m,p\}$ uning yo‘naltiruvchi vektori.

Ikki ayqash $\frac{x-x_1}{l_1}=\frac{y-y_1}{m_1}=\frac{z-z_1}{p_1}$ va $\frac{x-x_2}{l_2}=\frac{y-y_2}{m_2}=\frac{z-z_2}{p_2}$ to‘g‘ri chiziqlar orasidagi eng qisqa d masofa quyidagicha aniqlanadi: $d=\frac{|\overrightarrow{M_1M_2}\cdot\overrightarrow{s_1}\cdot\overrightarrow{s_2}|}{|\overrightarrow{s_1}\cdot\overrightarrow{\times}|}$ bunda $M_1(x_1,y_1,z_1)$ va $M_2(x_2,y_2,z_2)$ nuqtalar mos ravishda bu to‘g‘ri chiziqlarga tegishli, $\overrightarrow{s_1}=\{l_1,m_1,p_1\}$ va $\overrightarrow{s_2}=\{l_2,m_2,p_2\}$ lar esa ularning yo‘naltiruvchi vektorlari.

Asosiy qism. 1-Misol. $x-2y+2z-8=0$ va $x+z-6=0$ tekisliklar orasidagi burchakni toping.

Yechish: Ikki tekislik orasidagi burchak formulasiga ko‘ra:

$$\cos\varphi=\frac{A_1A_2+B_1B_2+C_1C_2}{\sqrt{{A_1}^2+{B_1}^2+{C_1}^2}\cdot\sqrt{{A_2}^2+{B_2}^2+{C_2}^2}}=\frac{1\cdot1+(-2)\cdot0+2\cdot1}{\sqrt{1+4+4}\cdot\sqrt{1+1}}=\frac{\sqrt{2}}{2}$$

Bundan $\varphi=arc\ cos\frac{\sqrt{2}}{2}=45^0$ kelib chiqadi.

$Ax+By+Cz+D=0$ tekislik bilan $\frac{x-x_0}{l}=\frac{y-y_0}{m}=\frac{z-z_0}{p}$ to‘g‘ri chiziq orasidagi φ burchak ushbu formula bo‘yicha hisoblanadi:

$$\sin\varphi=\frac{\vec{n}\cdot\vec{s}}{|\vec{n}|\cdot|\vec{s}|}=\frac{Al+Bm+Cp}{\sqrt{A^2+B^2+C^2}\cdot\sqrt{l^2+m^2+p^2}}$$

Bunda $\vec{n}=\{A,B,C\}-$ tekislikning normal vektori, $\vec{s}=\{l,m,p\}$ – to‘g‘ri chiziqning yo‘naltiruvchi vektori.

a) Agar tekislik bilan to‘g‘ri chiziq perpendikulyar bo‘lsa, $\vec{n}$ va $\vec{s}$ vektorlar kollinear yoki $\frac{A}{l}=\frac{B}{m}=\frac{C}{p}$ bo‘ladi.

b) Agar tekislik bilan to‘g‘ri chiziq parallel bo‘lsa, u holda $\vec{n}$ va $\vec{s}$ vektorlar perpendikulyar yoki $Al+Bm+Cp\neq 0$

c) Agar tekislik bilan to‘g‘ri chiziq ustma-ust tushsa, u holda

$Al+Bm+Cp=0$, shu bilan birga $Ax_0+By_0+Cz_0+D=0$ bo‘ladi.

d) Agar tekislik bilan to‘g‘ri chiziq kesishsa, u holda

$Al+Bm+Cp\neq 0$

Tekislikdagi to‘g‘ri chiziqlar: $A_1x+B_1y+C_1=0$ va $A_2x+B_2y+C_2=0$ tenglamalar bilan berilgan bo‘lsin. Ular orasidagi φ burchak ushbu formula bo‘yicha hisoblanadi:

$$\cos\varphi=\frac{\overrightarrow{n_1}\cdot\overrightarrow{n_2}}{|\overrightarrow{n_1}|\cdot|\overrightarrow{n_2}|}=\frac{A_1A_2+B_1B_2}{\sqrt{{A_1}^2+{B_1}^2}\cdot\sqrt{{A_2}^2+{B_2}^2}}$$

bunda $\overrightarrow{n_1}=\{A_1,B_1\}$, $\overrightarrow{n_2}=\{A_2,B_2\}$, – mos ravishda berilgan to‘g‘ri chiziqlarning normal vektorlari.

a) Agar bu to‘g‘ri chiziqlar o‘zaro perpendikulyar bo‘lsa, u holda

$$A_1\cdot A_2+B_1\cdot B_2=0$$

b) Agar bu to‘g‘ri chiziqlar parallel bo‘lsa, u holda $\frac{A_1}{A_2}=\frac{B_1}{B_2}$

c) Agar bu to‘g‘ri chiziqlar ustma-ust tushsa, u holda $\frac{A_1}{A_2}=\frac{B_1}{B_2}=\frac{C_1}{C_2}$

Tekislikdagi to‘g‘ri chiziqlar $y=k_1x+b_1$ va $y=k_2x+b_2$ tenglamalar bilan berilgan bo‘lsin. Ular orasidagi φ burchak ushbu formula bo‘yicha hisoblanadi: $\operatorname{tg}\varphi=\frac{k_2-k_1}{1+k_1.k_2}$

2-Misol. To‘g‘ri chiziqning yo‘naltiruvchi kosinusi topilsin.

1) $\frac{x-1}{4}=\frac{y-5}{-3}=\frac{z+2}{12}$

Bu chiziq (1,5,-2) nuqta orqali o‘tadi. Yo‘nalish vektori: $\vec{v}=(4,-3,12)$

Yo‘naltiruvchi kosinuslar – bu yo‘nalish vektorining koordinatalarining, uning uzunligiga nisbati: $\cos\alpha=\frac{a}{|\vec{v}|}$, $\cos\beta=\frac{b}{|\vec{v}|}$, $\cos\gamma=\frac{c}{|\vec{v}|}$ bu yerda $\vec{v}=(a,b,c)=(4,-3,12)$ $|\vec{v}|=\sqrt{4^2+(-3^2)+12^2}=13$

Javob : $\cos\alpha=\frac{4}{13}$, $\cos\beta=\frac{-3}{13}$, $\cos\gamma=\frac{12}{13}$

1) $\frac{x}{12}=\frac{y-7}{9}=\frac{z+3}{20}$

Bu chiziq (0,7,-3) nuqta orqali o‘tadi.

$\vec{v}=(a,b,c)=(12,9,20)$

$$|\vec{v}|=\sqrt{144+81+400}=25$$

Javob : $\cos\alpha=\frac{a}{|\vec{v}|}=\frac{12}{25}$, $\cos\beta=\frac{b}{|\vec{v}|}=\frac{9}{25}$, $\cos\gamma=\frac{c}{|\vec{v}|}=\frac{4}{25}$

3-Misol. $A(1,-5,3)$ nuqtadan o‘tib, koordinata o‘qlari bilan 60^0, 45^0, 120^0 burchak hosil qiluvchi to‘g‘ri chiziq tenglamasi tuzilsin.

Yechish: 1) To‘g‘ri chiziq x o‘qi bilan 60^0 burchak hosil qiladi:

$\cos\alpha = cos60^0 = \frac{l}{|\vec{v}|} = \frac{1}{2}$ $\vec{v} = (l,m,n)$

To‘g‘ri chiziq y o‘qi bilan 45^0 burchak hosil qiladi. $\cos\beta = cos45^0 = \frac{m}{|\vec{v}|} = \frac{\sqrt{2}}{2}$

To‘g‘ri chiziq z o‘qi bilan 120^0 burchak hosil qiladi: $\cos\gamma = cos120^0 = \frac{n}{|\vec{v}|} = \frac{-1}{2}$

bundan $\vec{v}(1,\sqrt{2},-1)$ $|\vec{v}| = 2$ ekanini bilishimiz mumkin.

1) $\frac{x-x_0}{l} = \frac{y-y_0}{m} = \frac{z-z_0}{n}$ to‘g‘ri chiziq tenglamasiga olib borib qo‘yamiz. (x_0,y_0,z_0) nuqta A(1,-5,3) bo‘ladi. $\frac{x-1}{1} = \frac{y+5}{\sqrt{2}} = \frac{z-3}{-1}$

Javob: $x-1 = \frac{y+5}{\sqrt{2}} = -(z-3)$

4-Misol. Ikki $\begin{cases}3x-4y-2z=0\\2x+y-2z=0\end{cases}$ va $\begin{cases}4x+y-6z-2=0\\y-3z+2=0\end{cases}$ to‘g‘ri chiziqlar orasidagi burchak topilsin.

Yechish: 1) $\begin{cases}3x-4y-2z=0\\2x+y-2z=0\end{cases}$ tekisliklar kesishishidan hosil bo‘lganto‘g‘ri chiziq tenglamasini tuzamiz: $\frac{x-x_0}{\begin{vmatrix}B_1 & C_1\\B_2 & C_2\end{vmatrix}} = \frac{y-y_0}{\begin{vmatrix}C_1 & A_1\\C_2 & A_2\end{vmatrix}} = \frac{z-z_0}{\begin{vmatrix}A_1 & B_1\\A_2 & B_2\end{vmatrix}}$

$(x_0,y_0,z_0) \to (0,0,0)$ nuqta bo‘ladi. A_1=3 A_2=2 B_1=-4 B_2=1 C_1=C_2=-2

$$\frac{x}{\begin{vmatrix}-4 & -2\\1 & -2\end{vmatrix}} = \frac{y}{\begin{vmatrix}-2 & 3\\-2 & 2\end{vmatrix}} = \frac{z}{\begin{vmatrix}3 & -4\\2 & 1\end{vmatrix}}$$

$$\frac{x}{10} = \frac{y}{2} = \frac{z}{11}$$

2) $\begin{cases}4x+y-6z-2=0\\y-3z+2=0\end{cases}$ tekisliklar kesishishidan hosil bo‘lganto‘g‘ri chiziq tenglamasini tuzamiz:

A_1=4 A_2=0 B_1=1 B_2=1 C_1=-6 C_2=-3

$$\frac{x-x_1}{\begin{vmatrix}1 & -6\\1 & -3\end{vmatrix}} = \frac{y-y_1}{\begin{vmatrix}-6 & 4\\-3 & 0\end{vmatrix}} = \frac{z-z_1}{\begin{vmatrix}4 & 1\\0 & 1\end{vmatrix}}$$

$$\frac{x-x_1}{3} = \frac{y-y_1}{12} = \frac{z-z_1}{4}$$

$x_1 = 0$ bo‘lganda, $-\begin{cases}y-6z=2\\y-3z=-2\end{cases} \Rightarrow -3z=4 \Rightarrow z=-\frac{4}{3}$

$y = 2+6z = 2-\frac{6\cdot4}{3} = -6$ $(0,-6,-\frac{4}{3})$

Demak, $\frac{x}{3} = \frac{y+6}{12} = \frac{y+\frac{4}{3}}{4}$

2) Endi 2 ta to‘g‘ri chiziq orasidagi burchakni topamiz: $\overrightarrow{v_1} = (10,2,11)$ $\overrightarrow{v_2} = (3,12,4)$

$$\cos\varphi = \frac{\overrightarrow{v_1}\cdot\overrightarrow{v_2}}{|\overrightarrow{v_1}|\cdot|\overrightarrow{v_2}|} = \pm\frac{30+24+44}{15\cdot13} = \pm\frac{98}{195}$$

$|\overrightarrow{v_1}| = \sqrt{100+4+21} = 15$ $|\overrightarrow{v_2}| = \sqrt{9+144+16} = 13$

Javob: $\cos\varphi = \pm\frac{98}{195}$

5-Misol. $\begin{cases} 2x+y-z+4=0 \\ x+y=0 \end{cases}$ fazoda bitta to‘g‘ri chiziqning OXZ tekislikdagi proyeksiya tenglamasi tuzilsin.

Yechish: 1) $\begin{cases} 2x+y-z+4=0 \\ x+y=0 \end{cases}$ fazoda bitta to‘g‘ri chiziqni ifodalaydi:

$$\frac{x-x_0}{\begin{vmatrix} B_1 & C_1 \\ B_2 & C_2 \end{vmatrix}} = \frac{y-y_0}{\begin{vmatrix} C_1 & A_1 \\ C_2 & A_2 \end{vmatrix}} = \frac{z-z_0}{\begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix}}$$

$A_1=2$ $A_2=1$ $B_1=1$ $B_2=1$ $C_1=-1$ $C_2=0$

(x_0, y_0, z_0) nuqtani topamiz. $x_0=0$ bo‘lganda $\begin{cases} y-z+4=0 \\ y=0 \end{cases} \Rightarrow$ z=4

Demak, (0,0,4) $\frac{x}{\begin{vmatrix} -1 & -1 \\ 1 & 0 \end{vmatrix}} = \frac{y}{\begin{vmatrix} -1 & 2 \\ 0 & 1 \end{vmatrix}} = \frac{z-4}{\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix}} \Rightarrow \frac{x}{1} = \frac{y}{-1} = \frac{z-4}{1}$

$x=-y=z-4$ *to‘g‘ri chiziq ekan.*

2) OXZ tekislikda y=0 bo‘ladi. Shuning uchun to‘g‘ri chiziqning OXZ proyeksiyasi $y=0,\ x-z+4=0$ bo‘ladi.

6-Misol. (-1,0,4) nuqtadan $x=1+t,\ y=2t,\ z=4-t$ to‘g‘ri chiziqqa tushirilgan perpendikulyarning tenglamasi tuzilsin.

Yechish: Nuqtadan to‘g‘ri chiziqqacha eng yaqin nuqtani topamiz. (perpendikulyar tushadigan nuqta).

P(t)=(1+t, 2t, 4-t) M(-1,0,4) berilgan nuqta. $\overrightarrow{MP}=(t+2, 2t, -t)$

Bu vektor, to‘g‘ri chiziqning yo‘nalish vektoriga perpendikulyar bo‘lishi kerak.

Yo‘nalish vektori: $\vec{v}=(1,2,-1)$

$$\overrightarrow{MP}\cdot\vec{v}=0$$
$$t+2+2t\cdot 2+t=0$$
$$6t+2=0$$
$$t=-\frac{1}{3}$$

Demak, P$(\frac{2}{3}, \frac{-2}{3}, \frac{13}{3})$ ekan.

2) Endi M(-1,0,4) va P$(\frac{2}{3}, \frac{-2}{3}, \frac{13}{3})$ nuqtalar orqali o‘tuvchi to‘g‘ri chiziq tenglamasini yozamiz:

$\frac{x+1}{\frac{2}{3}+1} = \frac{y}{-\frac{2}{3}} = \frac{z-4}{\frac{13}{3}-4} \Rightarrow \frac{x+1}{5} = \frac{y}{-2} = \frac{z-4}{1}$ to‘g‘ri chiziq ekan.

Javobni 2 ta tekislik ko‘rinishida ifodalaymiz.

$y=-2z+8 \Rightarrow y+2z-8=0$ va $-\begin{cases} 5y=-2x-2 \\ y=-2z+8 \end{cases} \Rightarrow$

$\Rightarrow 4y=-2x+2x-8 \Rightarrow x+2y-z+5=0$

$$Javob: \begin{cases} y+2z-8=0 \\ x+2y-z+5=0 \end{cases}$$

7-Misol. (1,-2,3) nuqtaning $6x-y+3z-41=0$ tekislikka nisbatan simmetrik nuqtasi topilsin.

Yechish: $x_1=1$ $y_1=2$ $z_1=-3$ $M_0(x_0, y_0, z_0)$ nuqtadan $Ax+By+Cz+D=0$ tekislikka tushirilgan perpendikulyarning tenglamasi

$$\begin{cases} x = x_0 + At \\ y = y_0 + Bt \\ z = z_0 + Ct \end{cases} \Rightarrow \begin{cases} x = 1 + 6t \\ y = 2 - t \\ z = -3 + 3t \end{cases}$$

$$\frac{x-x_1}{x_0-x_1} = \frac{y-y_1}{y_0-y_1} = \frac{z-z_1}{z_0-z_1} \Rightarrow \frac{x-1}{6} = \frac{y-2}{-1} = \frac{z+3}{3} \Rightarrow$$

$$x_0 - 1 = 6, \quad y_0 - 2 = -1, \quad z_0 + 3 = 3$$

$$\boldsymbol{x_0 = 7} \qquad \boldsymbol{y_0 = 1} \qquad \boldsymbol{z_0 = 0}$$

Javob: (7,1,0)

Xulosa qilib aytganda, tekisliklar va to‘g‘ri chiziqlarning o‘zaro joylashuvini o‘rganish analitik geometriyaning asosiy bo‘limlaridan biri hisoblanadi. Mazkur maqolada tekisliklar va to‘g‘ri chiziqlar orasidagi burchaklarni topish, ularning o‘zaro parallel, perpendikulyar, kesishuvchi va ayqash holatlarini aniqlash usullari ko‘rib chiqildi. Shuningdek, nuqtadan tekislik va to‘g‘ri chiziqqacha bo‘lgan masofalarni hisoblash, yo‘naltiruvchi kosinuslarni aniqlash, proyeksiyalar hamda simmetrik nuqtalarni topish masalalari misollar yordamida tahlil qilindi. Keltirilgan formulalar va yechimlar fazoviy geometrik obyektlarning xossalarini chuqurroq anglashga hamda amaliy masalalarni samarali hal etishga xizmat qiladi. Olingan natijalar analitik geometriya kursini o‘zlashtirishda va matematik modellashtirishga oid masalalarni yechishda muhim nazariy hamda amaliy ahamiyatga ega.

Foydalanilgan adabiyotlar

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